Fibonacci Conference

Book of abstracts

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Plenary talk: Exponential Diophantine equations with Fibonacci numbers

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Abstract

In my talk, I will survey some results obtained over the years with students and colleagues concerning Diophantine equations with Fibonacci numbers. For example, there is no perfect Fibonacci number. The largest pair Fibonacci numbers $\{F_m, F_n\}$ whose indices have the same parity and whose sum is a perfect power is $F_{36} + F_{12} = 3864^2$. The largest Fibonacci number which is a rep-digit is $F_{10} = 55$ and the largest Fibonacci number which is a concatenation of two rep-digits is $F_{14} = 377$. There is no Diophantine quadruple of Fibonacci numbers, namely set of four positive integers $\{a, b, c, d\}$ such that the product of any two plus 1 is a square and such that all are Fibonacci numbers. We will also present some newer results concerning the solutions of the equation $\sum_{j=1}^{k} jF_j^p = F_n^q$ in positive integers (k, n, p, q) as well as a result concerning elliptic curves over finite fields with Fibonacci numbers of points.

Plenary talk: A visual tour of Fibonacci numbers and their eccentric cousins, elliptic divisibility sequences

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Abstract

Fibonacci numbers have an eccentric cousin, elliptic divisibility sequences. These satisfy a non-linear recurrence, but from a geometric perspective, they both have their sources in certain groups: the multiplicative group for Fibonacci and Lucas sequences, and elliptic curves for elliptic divisibility sequences. I'll explore some of the familial similarities and differences and applications, and I'll take the opportunity to introduce a mathematical tool in development called Numberscope, for visualizing integer sequences.

A new combinatorial interpretation of the Fibonacci numbers cubed

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Abstract

We consider the tiling of an n-board (a $1\times n$ array of square cells of unit width) with third-squares ($\frac{1}{3}\times 1$ tiles) and $(\frac{1}{3}, \frac{2}{3})$-fence tiles. A $(\frac{1}{3}, \frac{2}{3})$-fence tile is composed of two third-squares separated by a gap of width $\frac{2}{3}$. We show that the number of ways to tile an n-board using these types of tiles equals F_{n+1}^3 where F_n is the nth Fibonacci number. We use these tilings to devise combinatorial proofs of identities relating the Fibonacci numbers cubed to one another and to other number sequences. Some of these identities appear to be new. This is joint work with Kenneth Edwards.

The Fibonacci Word as a 2-adic Number and its Continued Fraction

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Abstract

The infinite Fibonacci word, $\ldots 0110110101101$, considered as a 2-adic integer, is the limit of fixed points of a Fibonacci-like recursively defined sequence of linear functions. These fixed points, and their limit, have "remarkable continued fractions" of the form $-\frac{2^0}{1+}\ \frac{2^1}{1+}\ \frac{2^1}{1+}\ \frac{2^2}{1+}\ \frac{2^3}{1+}\ \cdots\ \frac{2^{F_n}}{1+}\ \cdots$. Previous work showed the Fibonacci word $1011010110110\ldots$ as a traditional number (in the Euclidean metric) between 0 and 1 (i.e., preceded by "0.") has continued fraction $\frac{1}{2^0+}\ \frac{1}{2^1+}\ \frac{1}{2^1+}\ \frac{1}{2^2+}\ \frac{1}{2^3+}\ \cdots\ \frac{1}{2^{F_n}+}\ \cdots$.

Lyndon words and second-order recurrences

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Abstract

A Lyndon word is a word that is primitive (not a power of another word) and earlier in lexical or numerical order than any of its cyclic shifts. The talk will survey Lyndon words generated by an l-order linear recurrence (mod p) having unit coefficients, where $l = 2$ and p is a prime. This type of recurrence produces Lyndon words that include the Fibonacci sequence (mod p).

Consider the set of Lyndon words of length two formed using the digits from 0 to $p-1$. Using these words as starting sequences of the recurrence, one finds that some of them generate Lyndon words of length equal to the least period of the recurrence (mod p). It's natural to ask: what fraction of the starting sequences do so? Analysis of the characteristic polynomial $f(x)$ associated with the recurrence shows this fraction to be λ/p, where λ is a positive integer and $f(x)$ is irreducible. When $f(x)$ is reducible, the fraction asymptotically approaches λ/p with increasing p. An exact relation that covers both cases will be presented. λ appears to be a basic parameter of the recurrence (mod p) and its value serves to identify the least periods that occur.

Norm Form Equations and Linear Divisibility Sequences

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Abstract

Finding integer solutions to norm form equations is a classic Diophantine problem. Using the units of the associated coefficient ring, we can produce sequences of solutions to these equations. It turns out that such a sequence can be written as a tuple of integer linear recurrence sequences, each with characteristic polynomial equal to the minimal polynomial of our unit. We show that in some cases, these sequences are linear divisibility sequences.

Geometry in the Determinant Hosoya Triangle

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Abstract

The Determinant Hosoya triangle is a triangular array of integers in which each entry is formed by taking a determinant of a matrix comprised of Fibonacci numbers. Additionally, these entries are also generated recursively, similar to Pascal's triangle, in which each entry is the sum of the two entries in the either upper left or upper right diagonals. These recursive and determinant properties lead to a plethora of interesting results when using well-known linear algebra techniques and taking a geometric approach to the triangle. We have formed strong connections between well-known Fibonacci and Lucas identities alongside connections with the original Hosoya's Triangle. This is a joint work with R. Florez and A. Mukherjee.

A Pellian equation with primes and its applications

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Abstract

The Pell numbers are numbers defined by the recurrence relation

$$P_0 = 0,\ P_1 = 1,\ P_{n+1} = 2P_n + P_{n-1},\ n \geq 1.$$

Those numbers are 2-Fibonacci numbers and they are closely related to solutions of the Diophantine equations $x^2 - 2y^2 = \pm 1$: if (x, y) denote a non-negative integer solution of any of these equation, then $y = P_i$, for some non-negative integer i. The Diophantine equation $x^2 - dy^2 = n$, where d is is a positive non-square integer and n is a non-zero integer is called Pellian equation. In this talk we will briefly describe main results concerning of a solution of such equations and present results on the solubility of the Pellian equation $x^2 - (p^{2k+2} + 1)y^2 = -p^{2\ell+1}$, $\ell \in \{0, 1, \ldots, k\}$, $k \geq 0$, (1) with a prime p. We will apply the obtained results on the Diophantine m-tuple problem.

A set of m non-zero elements $a_1, \ldots, a_m$ of of a commutative ring R is a Diophantine m-tuple with the property $D(-1)$ or just a $D(-1) - m$-tuple if $a_i a_j - 1$ is a perfect square in R for all i and j with $1 = i < j = m$. The existence of a positive integer solutions of the equation (1) is related to the existence of some $D(-1)$-quadruples in a certain ring. By combining results about the solubility of (1) with other known results on the existence of the Diophantine quadruples, we will present results on the extensibility of some parametric families of $D(-1)$-pairs (for example, pairs which contain Fermat prime) to quadruples in the ring $\mathbb{Z}[\sqrt{-t}], t > 0$.

Completeness of Positive Linear Recurrence Sequences

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Abstract

A sequence of positive integers is called complete if every positive integer can be written as a sum of distinct terms of the sequence. We examine completeness phenomena for positive linear recurrence sequences, which arise as natural generalizations of the Fibonacci numbers when studying generalized Zeckendorf decompositions. We use Brown's Criterion to determine completeness in specific cases, and establish new criteria for completeness using the principal root of the characteristic polynomial of the recurrences. This is joint work with John Haviland, John Lentfer, Phuc Lam, Steven J. Miller and Fernando Trejos.

Geometric Capitulum Patterns based on Golden p-Angles

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Abstract

Modeling based on the golden angle has provided insight into how densely packed phyllotaxis structures and organizational patterns arise. The classic example of a pattern produced by the golden angle is the organization of florets on a sunflower. Studies of geometric pattern generation shows that structure organization and covering is highly sensitive to the angle separating the individual primordia which arise in a meristem with the Fibonacci angle giving the parastichy with optimal packing density. In other words, packing efficiencies and organization of a covering of a geometric capitulum are modeled according to angle of rotation which produces a circular pattern. Angles that are a rational fraction of a turn usually result in very poor coverings while the golden angle produces one of the best coverings. To begin to understand how other phyllotaxis patterns might arise, we generated different geometric patterns based on the generalized golden p-sections, which are linked to the p-Fibonacci numbers. In our modeling, the golden p-angle is the angle subtended by the smallest arc when the arcs that make up a circle are modeled according to the generalized golden p-sections. That is, the golden p-angle is the smaller of the angles created by sectioning the circumference of a circle based on the golden d-ratio, where $d > 1$. In the case of $d = 1$, ϕ is the golden ratio. Generation of various geometric structures shows that different efficiencies of covering and regular organizational patterns occur across different golden p-angles. Conclusion: studying geometric capitulum patterns based on golden p-angles begins to show how the regularity in phyllotaxis patterns might occur in biology.

Gaps Of Summands of The Zeckendorf Lattice

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Abstract

A beautiful theorem of Zeckendorf states that every positive integer has a unique decomposition as a sum of non-adjacent Fibonacci numbers. Such decompositions exist more generally, and much is known about them. First, for any positive linear recurrence $\{G_n\}$ the number of summands in the legal decompositions for integers in $[G_n, G_{n+1} >$ converges to a Gaussian distribution. Second, Bower, Insoft, Li, Miller, and Tosteson proved that the probability of a gap between summands in a decomposition which is larger than the recurrence length converges to geometric decay. While most of the literature involves one-dimensional sequences, some recent work by Chen, Guo, Jiang, Miller, Siktar, and Yu have extended these decompositions to d-dimensional lattices, where a legal decomposition is a chain of points such that one moves in all d dimensions to get from one point to the next. They proved that some but not all properties from 1-dimensional sequences still hold. We continue this work and look at the distribution of gaps between terms of legal decompositions, and prove similar to the 1-dimensional cases that when $d = 2$ the gap vectors converge to a bivariate geometric random variable. This is joint work with Bruce Fang, Steven J. Miller and Wanqiao Xu.

Fibonacci numbers and real quadratic p-rational fields

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Abstract

We characterize p-rational real quadratic fields in terms of generalized Fibonacci numbers by using congruences of special valuesof L-functions. We then use this characterization to give numerical evidence to a conjecture of Greenberg asserting the existence of p-rational multiquadratic fields of arbitrary degree 2^t, $t \geq 1$.

Diophantine equations with Fibonacci and generalized Pell numbers

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Abstract

The Pell sequence $(P_n)_{n\geq 0}$ is the second order linear recurrence defined by $P_n = 2P_{n-1} + P_{n-2}$ with initial conditions $P_0 = 0$ and $P_1 = 1$. In this talk, we present some recent work on a generalization of the Pell sequence called the k-Pell sequence $(P_n^{(k)})_n$ which is generated by a recurrence relation of a higher order. We report about some arithmetic properties of $(P_n^{(k)})_n$ and study some Diophantine equations involving Fibonacci and k-Pell numbers. This is a joint work with Florian Luca and Jose L. Herrera.

The weak converse of Zeckendorf's Theorem

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Abstract

Zeckendorf's Theorem allows us to write each positive integer uniquely as a sum of distinct non-consecutive Fibonacci terms. The uniqueness is an interesting feature of the representation, but, arguably, its converse is rather more interesting; the Fibonacci sequence is the only sequence of positive integers that uniquely represents each positive integer in such a fashion. If we further impose a monotone property on the sequences, we call it the weak converse of Zeckendorf's theorem. In this talk, we introduce results on the weak converse of Zeckendorf's theorem for the standard Zeckendorf condition associated with a general linear recurrence and for some nonperiodic Zeckendorf conditions, called f-decomposition. We introduce results for the real numbers as well as the positive integers.

On the relation between Fibonacci and Lucas numbers of order k

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Abstract

Let $(F_n^{(k)})_{n\geq 0}$ be the sequence of Fibonacci numbers of order k and $(L_n^{(k)})_{n\geq 0}$, be the sequence of Lucas numbers of order k. There are several well-known identities in the literature revealing the relation between Fibonacci and Lucas numbers of order k. We present some of these identities employing the combinatorial interpretations of the numbers, or their generating functions. We finally prove the identity

$$\sum_{i=0}^{n} (-1)^i m^{n-i} \left(L_{i+1}^{(k)} + (m-2)\, F_i^{(k)} \sum_{j=3}^{k} j F_{i-j+2}^{(k)} \right) = (-1)^n F_{n+1}^{(k)}. \quad (2)$$

Three problems with solutions involving Fibonacci and Pell numbers

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Abstract

Three independent distinct problems were set and solved, in particular:

1. To express π as the sum of two summands each with infinite terms involving reciprocals of Fibonacci (or Pell) numbers, the first being equal to $2\tan^{-1}\alpha$ and the second being equal to $2\tan^{-1}(1/\alpha)$, where $\alpha = \frac{1+\sqrt{5}}{2}$;
2. To find new formulas determining infinite products involving Pell numbers;
3. To find a criterion when an integer M is a Pell number.

Some new formulas, identities and propositions were obtained. Short analysis and evaluation of the resulting formulas were made as well.

Linear independence of infinite products involving Lucas numbers

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Abstract

The main subject of this talk is the following result. Let (L_n) be the sequence of Lucas numbers. Then the numbers1, $\prod_{n=1}^{\infty}\left(1+\frac{1}{L_{2n}}\right)$, $\prod_{n=1}^{\infty}\left(1-\frac{1}{L_{2n}}\right)$, $\prod_{n=1}^{\infty}\left(1+\frac{2}{L_{2n}}\right)$ and $\prod_{n=1}^{\infty}\left(1-\frac{2}{L_{2n}}\right)$ are linearly independent over $\mathbb{Q}\left(\sqrt{5}\right)$. In particular, each of the four last numbers is irrational. The proof of this result rests on beautiful properties of the function

$$H_q(x)=\prod_{n=1}^{\infty}\left(1+\frac{q^n x}{q^{2n}+1}\right)$$

$(|q|<1,\ x\in\mathbb{C})$, introduced by Jean-Paul Bézivin, and on an elementary criterion of irrationality for certain gap series. Joint work with Yohei Tachiya

A connection between Fibonacci Identities and Abstract Algebra

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Abstract

In this talk we give a connection between Fibonacci numbers and the abstract algebra. We give a series of matrices, from literature, used to obtain recurrence relations of second order, third order and polynomial sequences. We also give a list of some known identities in algebra. Using the mentioned matrices as a bridge we obtain a series of new and classical identities in Fibonacci numbers, Lucas numbers, Pell numbers, binomial transform, tribonacci numbers, and polynomial sequences. The list of identities that we give in this talk is actually a few examples out of many that are possible to find using this technique. This is joint work with Santiago Alzate, Oscar Correa, and Kevin Lopez.

Recursive Triangles Appearing Embedded in Recursive Families

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Abstract

We say a triangle appears embedded in a recursive sequence $\{G_i\}_{i\geq 1}$ if there are integers s, r, c such that when the subsequence $G_{s+1}, \ldots, G_{s+rc}$ is laid out in an $r \times c$ rectangle, the indices of the last non-zero item in each row are strictly increasing. Two sequences of different orders with appeared embedded triangles are compatible if they agree on all non-zero items in shared rows. An embedded triangle appears embedded in a family of recursive sequences if triangles appear embedded in all members and the members are compatible. We associate recursive sequences with their characteristic polynomials and associate a Taylor series with the family of recursive sequences associated with the approximating Taylor polynomials We prove one main result: A triangle appears embedded in a subfamily of the recursive family associated with $F(x) = \frac{1}{1-x} + \sum_{i=1}^{q}(m_i - 1)x^{p_i - 1} - 2$, for integers, $q \geq 1, m_i \geq 2, 1 \leq i \leq q, 1 < p_1 < p_2 < \ldots < p_q$. The embedded triangle satisfies a triangle recursion. We completely describe the family, the recursions, and all associated parameters. We conclude by presenting numerical evidence for the Hendel minimum absolute-value conjecture.

On a generalization of the Pell sequence

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Abstract

The Pell sequence $(P_n)_{n=0}^{\infty}$ is the second order linear recurrence defined by $P_n = 2P_{n-1} + P_{n-2}$ with initial conditions $P_0 = 0$ and $P_1 = 1$. In this talk, we present some recent work on a generalization of the Pell sequence called the k-generalized Pell sequence which is generated by a recurrence relation of a higher order. We give recurrence relations, a generalized Binet formula and different arithmetic properties for the above family of sequences. Some interesting identities involving the Fibonacci and generalized Pell numbers are also presented. This is a joint work with Florian Luca and Jhon Jairo Bravo.

The Continued Fraction Pendulum

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Abstract

For an irrational number α and a positive integer n we consider the fractional parts of multiples $i\alpha$ for $i = 1, \ldots, n$. The Three Distance Theorem states that these n points partition the unit interval into gaps of at most three distinct lengths. My talk explains how the process of splitting gaps for increasing n swings like a pendulum in the margins of the unit interval.

Moreover, the metaphor provides an enticing approach to the continued fraction representation of α.

In addition to that, it might be the key for solving a generalization of Problem 2, cf. Proceedings of the Fibonacci Conference in Halifax, 2018.

Degenerate Eulerian polynomials and generalization of permutations

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Abstract

Permutations can be classified and enumerated in many different ways. An Eulerian number $A_{n,k}$ enumerates a number of permutations of length n with exactly k ascents. The Eulerian polynomials were generalized by Carlitz twice – in 1954, when he defined q-Eulerian polynomials, and in 1956, when he defined degenerate Eulerian polynomials. There is a combinatorial interpretation of coefficients of the q-Eulerian polynomials, however there is no publications about combinatorial aspects of the degenerate Eulerian polynomials. In this work we present a combinatorial interpretation of coefficients of the degenerate Eulerian polynomials.

Ties in worst-case analysis of the Euclidean algorithm

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Abstract

We determine all pairs of positive integers below a given bound that require the most division steps in the Euclidean algorithm. Also, we find asymptotic probabilities for a unique maximal pair or an even number of them. This refines and expands related work of Merkes and Meyer from 1973. Our primary tools are continuant polynomials and the Zeckendorf representation using Fibonacci numbers. This is joint work with Aram Tangboonduangjit of Mahidol University International College in Thailand.

Asymptotic Analysis For Lattice Walks Derived From Zeckendorf Decompositions

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Abstract

Zeckendorf's Theorem states that any positive integer can be written uniquely as a sum of non-adjacent Fibonacci numbers, which leads to an interesting alternative definition of the Fibonacci numbers as the unique integer sequence with this decomposition property. As $n \to \infty$, the distribution of the number of summands in the Zeckendorf decomposition of $m \in [F_n, F_{n+1})$ converges to a Gaussian. With the one dimensional case well studied, a natural generalization considers sequences indexed by higher dimensional lattices $\mathbb{Z}^d$. Paths along this lattice take the place of subsequences in a decomposition, with constraints on the shape of the paths replacing the non-adjacent summand constraint. As simple computations show that no sequence exists with unique path decompositions of the integers analogous to the one dimensional case, we investigate asymptotic results.

We study two cases: paths are either simple or unrestricted jump paths. The former has paths where every point chosen has each component strictly less than the component of the previous chosen point in the path; the latter relaxes the requirement to where each subsequent point must be strictly smaller in at least one coordinate. In both cases the distribution of the number of summands in these lattice decompositions converges to a Gaussian. In the case of simple jump paths, thanks to the simpler closed form combinatorial expressions that arise, we prove this result in all positive dimensions. In the unrestricted case, more complicated inclusion-exclusion and generating function arguments are required, so we only show Gaussianity in 2 dimensions, although a more involved analysis is expected to hold for higher dimensions. This is joint work with Ethan Lu, Steven J. Miller, Joshua M. Siktar and Peter Yu.

Terms of Binary Recurrence Sequences which are Products of Factorials

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Abstract

If $\{U_n\}_{n\geq 0}$ is a Lucas sequence, then the largest n such that $|U_n| = m_1!m_2!\cdots m_k!$ with $1 < m_1 \leq m_2 \leq \cdots \leq m_k$ satisfies $n < 65000$. In case the roots of the Lucas sequence are real, we have $n \in \{1, 2, 3, 4, 6, 12\}$. As a consequence, we show that if $\{X_n\}_{n\geq 1}$ is the sequence of $X-$coordinates of a Pell equation $X^2 - dY^2 = \pm 1$ with a nonzero integer $d > 1$, then $X_n = m!$ implies $n = 1$. This is a joint work with F. Luca and M. Sias.

Counting on Euler and Bernoulli Number Identities

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Abstract

While there are many identities involving the Euler and Bernoulli numbers, they are usually proved analytically or inductively. We prove two identities involving Euler and Bernoulli numbers with combinatorial reasoning via up-down permutations. This is joint with Arthur T. Benjamin and Thomas C. Martinez.

Generalizing Zeckendorf's Theorem to Homogeneous Linear Recurrences

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Abstract

Zeckendorf's theorem states that every positive integer can be written uniquely as the sum of non-consecutive Fibonacci numbers $\{F_n\}$, where we take $F_1 = 1$ and $F_2 = 2$; in fact, it provides an alternative definition of the Fibonacci numbers. This has been generalized for any Positive Linear Recurrence Sequence (PLRS), which is, informally, a sequence satisfying a homogeneous linear recurrence with a positive leading coefficient and non-negative integer coefficients. Note these legal decompositions are generalizations of base B decompositions. We investigate linear recurrences with leading coefficient zero, followed by non-negative integer coefficients, with differences between indices relatively prime (abbreviated ZLRR), via two different approaches. The first approach involves generalizing the definition of a legal decomposition for a PLRS found in Koloğlu, Kopp, Miller and Wang. We prove that every positive integer N has a legal decomposition for any ZLRR using the greedy algorithm. We also show that D_n / n, the average number of decompositions of all positive integers less than or equal to n, is greater than 1, implying the existence of decompositions for every positive integer N, but lack of uniqueness. The second approach converts a ZLRR to a PLRR that has the same growth rate. We develop the Zeroing Algorithm, a powerful helper tool for analyzing the behavior of linear recurrence sequences. We use it to prove a very general result that guarantees the possibility of conversion between certain recurrences, and develop a method to quickly determine whether our sequence diverges to $+\infty$ or $-\infty$, given any real initial values. Joint work with Chenyang Sun, Steven J. Miller, and Clay Mizgerd.

Matrices in the Determinant Hosoya Triangle

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Abstract

The *determinant Hosoya triangle* is a triangular array where each entry is the determinant of two-by-two Fibonacci matrices. In this presentation we discuss properties of square persymmetric and symmetric matrices embedded in the determinant Hosoya triangle. Specifically we present results on the rank, eigenvalues, eigenvectors, and characteristic polynomials of these matrices. We also show that some of these properties can

be expressed as closed formulas involving Fibonacci and Lucas numbers. This is joint work with Mathew Blair and Rigoberto Flórez.

An Identity for Inverse-Conjugate Compositions

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Abstract

We prove a combinatorial identity between two classes of inverse-conjugate compositions, that is, integer compositions whose conjugates are given by a reversal of their sequences of parts. These are the set of inverse-conjugate compositions of $2n+3$ without 2's, and the set of inverse-conjugate compositions of $2n-1$ with parts not exceeding 3. Both sets are enumerated by $2F_n$, where F_n is the nth Fibonacci number.

A Triangle with Sides Lengths of the Rational Power of Plastic Constant

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Abstract

Using the golden ratio ϕ, a triangle whose side length ratio can be expressed as $1 : \sqrt{\phi} : \phi$ represents a right triangle. Because, the golden ratio has the property of $\phi^2 = \phi + 1$. Since this satisfies $1^2 + (\sqrt{\phi})^2 = \phi^2$, it becomes a right triangle. This triangle is called the Kepler triangle. This study, like Kepler triangle, we determines a triangle where the length of the 3 side of the triangle is expressed using only a constant (Golden Ratio, Plastic Constant, Tribonacci Constant, Supergolden Ratio) obtained from a linear regression sequence.

Topographs; Conway and otherwise

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Abstract

The 3-regular tree can be embedded in the plane so as to define infinitely many faces. Assigning values (generally from a ring) that satisfy a simple local recurrence defines a "topograph". A particular type, the "Conway topograph", has shown itself to be a simple but powerful tool for understanding quadratic forms. In these, the Fibonacci numbers make a surprise appearance.

We consider Conway topographs as well as topographs in general, where the rule is allowed to vary. We note that topographs, being two-dimensional generalizations of sequences, can exhibit periodicity in various ways. We look at several examples.

Note on Brousseau's Summation Problem

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Abstract

This paper takes a historical view of some long-standing problems associated with the development of sums of Fibonacci numbers in which the latter have powers of integers as coefficients. The sequences of coefficients of these polynomials are arrayed in matrices with links to *The On-Line Encyclopedia of Integer Sequences*. This is an extension of previous work on the summation problem of Ledin because Brousseau introduced some elegant techniques and the papers of both authors link with some interesting matrices. This is joint work with Anthony G. Shannon.

A Combinatorial Interpretation for the Generalized Hoggatt Matrices

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Abstract

In 1936, Ward considered a generalization of the classical binomial coefficients associated to the sequence $\{u_n\}_{n\in\mathbb{N}}$, denoted and defined as $[n, r] = \frac{u_n u_{n-1} \cdots u_{n-r+1}}{u_1 u_2 \cdots u_r}$. For some particular cases of the sequence $\{u_n\}_{n\in\mathbb{N}}$, we recover sequences like the fibonomial coefficient and the q-binomial coefficient. The matrix defined by these coefficients has similar properties as the classical Pascal triangle. In this talk we give a connection between the Ward binomial coefficients and the generalized Hoggatt matrices defined by Fielder and Alford in 1989. In particular, we show that these matrices have an interesting combinatorial interpretation by means of non-intersecting lattice path. This result was derived from the theorem of Gessel-Viennot to evaluated binomial determinants. After that, we derive several combinatorial identities by using generating function and a symbolic calculus called h-calculus. This talk is based on a joint work with Ana Luzón and Manuel Morón.

On the l.c.m. of random terms of binary recurrence sequences

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Abstract

For every positive integer n and every $\delta \in [0, 1]$, let $B(n, \delta)$ denote the probabilistic model in which a random set $A \subseteq \{1, \dots, n\}$ is constructed by choosing independently every element of $\{1, \dots, n\}$ with probability δ. Moreover, let $(u_k)_{k \geq 0}$ be an integer sequence satisfying $u_k = a_1 u_{k-1} + a_2 u_{k-2}$, for every integer $k \geq 2$, where $u_0 = 0$, $u_1 \neq 0$, and a_1, a_2 are fixed nonzero integers; and let α and β, with $|\alpha| \geq |\beta|$, be the two roots of the polynomial $X^2 - a_1 X - a_2$. Also, assume that α/β is not a root of unity.

We prove that, as $\delta n / \log n \to +\infty$, for every A in $B(n, \delta)$ we have $\log \operatorname{lcm}(u_a : a \in A) \sim \frac{\delta \operatorname{Li}_2(1-\delta)}{1-\delta} \cdot \frac{3 \log\left|\alpha/\sqrt{(a_1^2, a_2)}\right|}{\pi^2} \cdot n^2$ with probability $1 - o(1)$, where lcm denotes the lowest common multiple, Li_2 is the dilogarithm, and the factor involving δ is meant to be equal to 1 when $\delta = 1$.

This extends previous results of Akiyama, Tropak, Matiyasevich, Guy, Kiss and Mátyás, who studied the deterministic case $\delta = 1$, and is motivated by an asymptotic formula for $\operatorname{lcm}(A)$ due to Cilleruelo, Rué, Šarka, and Zumalacárregui.

Random Fibonacci sequences from a Balancing word pattern

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Abstract

We study growth rates of random Fibonacci sequences of a particular structure. A random Fibonacci sequence is an integer sequence starting with $1, 1$ where the next term is determined to be either the sum or the difference of the two preceeding terms where the choice of taking either the sum or the difference is chosen randomly at each step. In 2012, McLellan proved that if the pluses and minuses follow a periodic pattern and G_n is the nth term of the resulting random Fibonacci sequence, then $\lim_{n\to\infty} |G_n|^{1/n}$ exists. We extend her results by showing that this limit also exist if the choices of pluses and minuses follow a balancing word pattern. This is joint work with Dr. Kevin Hare.

Symmetric functions and Fibonacci numbers

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Abstract

We propose new identities for some symmetric polynomials: elementary, complete homogeneous, power symmetric polynomials, which are relationships between symmetric polynomials in $2r$ variables z_1, z_1^{-1}, ..., z_r, z_r^{-1} and r variables $z_1 + z_1^{-1}$, ..., $z_r + z_r^{-1}$. As applications of these identities, we derive some formulas for a higher order analogue of Fibonacci and Lucas numbers.

The Markoff equation with Fibonacci components

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Abstract

The Markoff equation $x^2 + y^2 + z^2 = 3xyz$ has infinitely many solutions. The components of a Markoff triple (x, y, z) are called Markoff numbers. It is known that all odd Fibonacci numbers are Markoff numbers and $(1, F_n, F_{n+2})$ is a Markoff triple for all odd positive integers n. We present a brief history around this fascinating equation and give an outline of the proof of our main result, that any Markoff triple with all Fibonacci components is of the type mentioned above. This is joint work with Florian Luca (Florian.Luca@wits.ac.za, University of the Witwatersrand, South Africa).

Chromatic number of Fibonacci distance graphs

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Abstract

Let $D = \{F_2, F_3, \ldots, F_t\}$ where as usual, F_i denote the ith Fibonacci number. Then the Fibonacci distance graph $G_n(D)$ is a graph whose vertices are natural numbers $1, 2, 3, \ldots, n$ and two vertices x and y are adjacent whenever $|x - y| \in D$. We completely determine the vertex chromatic number $\chi(G_n(D))$ of all such distance graph $G_n(D)$.

Polynomials Associated to the Combinations of Powers of Cosine and Sine

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Abstract

By expressing the sum of powers of cosine and sine as a polynomial in the sum of cosine and sine, we obtain a sequence of rational polynomials into which several properties are investigated. One surprising result is a connection to expressing the terms of the Lucas sequence as a finite product of some trigonometric functions. This is joint work with Aram Tangboonduangjit.

Girard-Waring Type Formula For A Generalized Fibonacci Sequence

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Abstract

Let $f(x) = x^k + a_1 x^{k-1} + \cdots + a_k$ be a monic polynomial of degree $k \geq 2$ with distinct roots $\{x_i | i = 1, \ldots, k\}$. Let $f'(x)$ be the derivative of $f(x)$, $P_n = x_1^n/f'(x_1) + x_2^n/f'(x_2) + \cdots + x_k^n/f'(x_k)$ and $Q_n = x_1^n + x_2^n + \cdots + x_k^n$; P_n is a generalized Fibonacci sequence and Q_n is a generalized Lucas sequence. We have a Girard-Waring Type Formula for P_n:

$$P_n = \sum_{j_1, \ldots, j_k} (-1)^{(j_1+j_2+\cdots+j_k)} \cdot (a_1^{j_1} a_2^{j_2} \cdots a_k^{j_k}) \cdot \frac{(j_1 + j_2 + \cdots + j_k)!}{j_1! j_2! \cdots j_k!}$$

where the indices $j_1, j_2, \ldots, j_k$ satisfy $j_1 + 2j_2 + \cdots + kj_k = n - k + 1$. We have formulas for the generating function for P_n, and Q_n: $G_P(x) = (1/x)/f(1/x)$ and $G_Q(x) = (1/x)f'(1/x)/f(1/x)$; we also have identity for Q_n in terms of P_n.